STiCM

Select / Special Topics in Classical Mechanics

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STiCM Lecture 29

Unit 9 : **Fluid Flow, Bernoulli's Principle**

Curl of a vector, Fluid Mechanics / Electrodynamics, etc.

Unit 9: Fluid Flow, Bernoulli's Principle

Definition of circulation, *curl*, vorticity,

irrotational flow.

Steady flow.
Bernoulli's equation/principle, some illustrations.

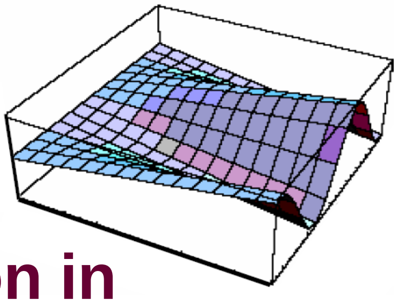
Unit 10: Fluid Flow, Bernoulli's Principle. Equation of motion for fluid flow. Definition of curl, vorticity, Irrotational flow and circulation. Steady flow. Bernoulli's principle, some illustrations. Introduction to applications of Gauss' law and Stokes' theorem in Electrodynamics.

Learning goals: Learn that both the divergence and the curl of a vector field are involved (along with the boundary conditions) in determining its properties.

Learn how a rigorous treatment of the velocity field is necessary to explain quantitatively the observed phenomena in fluid dynamics.

Get ready for a theory of electrodynamics.

Recall the discussion on directional derivative



$$\frac{d\psi}{ds} = \hat{u} \bullet \vec{\nabla}\psi$$

$$\hat{u} = \lim_{\delta s \to 0} \frac{\overrightarrow{\delta r}}{\delta s} = \frac{\overrightarrow{dr}}{ds}$$

$$\delta s = \left|\overrightarrow{\delta r}\right|, \text{ tiny increament}$$

$$ds = \left|\overrightarrow{dr}\right|, \text{ differential increament}$$

Gradient: direction in which the function varies fastest / most rapidly.

$$\vec{F} = -\vec{\nabla}\psi$$

Force: Negative gradient of the potential

'negative' sign is the result of our choice of natural motion as one occurring from a point of 'higher' potential to one at a 'lower' potential.

$\vec{F} = m\vec{a}$

Is it obvious that the 'force' defined by these two equations is essentially the same?

$\vec{F} = -\vec{\nabla}\psi$

Consistency in these relations exists only for 'conservative' forces.

Compatibility of the two expressions holds if, and only if, the potential ψ is defined in such way that the work done by the force given by $-\vec{\nabla}\psi$ in displacing the object on which this force acts, is independent of the path along which the displacement occurred.

$$\oint \vec{F} \bullet \vec{dr} = 0$$

$\int_a^b \vec{F} \bullet \vec{dr}$ *is INDEPENDENT of the path a to b*

PATH INTEGRAL
"CIRCULATION"

$$\oint \vec{F} \bullet \vec{dr} = 0$$

$$\int_a^b \vec{F} \bullet \vec{dr} \ is$$

INDEPENDENT of the path a to b

It is only when the line integral of the work done is path independent that the force is conservative and accounts for the acceleration it generates when it acts on a particle of mass m through the 'linear response' mechanism expressed in the principle of causality of Newtonian mechanics:

$$\vec{F} = m\vec{a}$$

The path-independence of the above line integral is completely equivalent to an alternative expression which can be used to define a conservative force.

This alternative expression employs what is known as CURL of a VECTOR FIELD $\vec{F}$, denoted as $\vec{\nabla} \times \vec{F}$.

Definition of Curl of a vector: $\vec{\nabla}\times\vec{F}$ is a vector point function of the vector field $\vec{F}(\vec{r})$ such that, for an orthonormal basis set of unit vectors $\{\hat{u}_i(\vec{r}), i = 1, 2, 3\}$,

$$\hat{u}_i(\vec{r}) \bullet \vec{\nabla}\times\vec{F}(\vec{r}) = \lim_{\Delta S \to 0} \frac{\oint \vec{F}(\vec{r}) \bullet \vec{dr}}{\Delta s},$$

Mean (average) 'circulation' per unit area taken at the point when the elemental area becomes infinitesimally small.

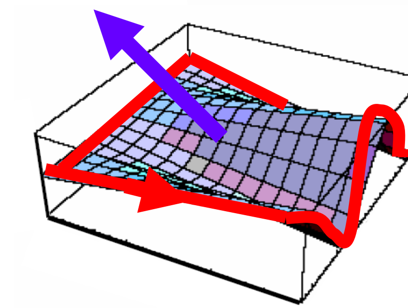
where the path integral is taken over a closed path C, taken over a tiny closed loop C which bounds an elemental vector *surface area* $\overrightarrow{\Delta S} = \Delta S\ \hat{u}_i(\vec{r})$.

The direction of the unit vector $\hat{u}_i(\vec{r})$ is such that a right-hand screw would propagate forward along it when it is turned along the sense in which the path integral is determined.

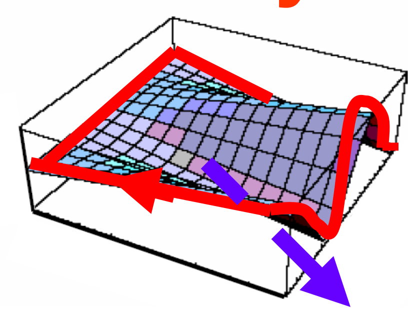
$$\hat{u}_i(\vec{r}) \bullet \vec{\nabla} \times \vec{F}(\vec{r}) = \lim_{\Delta S \to 0} \frac{\oint \vec{F}(\vec{r}) \bullet \overrightarrow{dr}}{\Delta s},$$

where the path integral is taken over a closed path C, taken over a tiny closed loop C which bounds an elemental vector *surface area*

$$\overrightarrow{\Delta S} = \Delta S \;\hat{u}_i(\vec{r}).$$



C traversed one way



C traversed the other way

right-hand-screw convention.

$$\hat{u}_i(\vec{r}) \bullet \vec{\nabla} \times \vec{F}(\vec{r}) = \lim_{\Delta S \to 0} \frac{\oint \vec{F}(\vec{r}) \bullet \overrightarrow{dr}}{\Delta s}$$

$$\left\{ \hat{u}_i(\vec{r}), i = 1, 2, 3 \right\}$$

In general, the unit vectors may depend on the particular point under discussion, and hence written as functions $\hat{u}_i(\vec{r})$ of $\vec{r}$.

Cartesian unit vectors, of course,
do not change from point to point.

They are constant vectors.

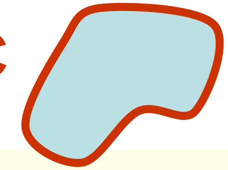
The above definition of *CURL of a VECTOR* is independent of any coordinate frame of reference; it holds good for any complete orthonormal set of basis set of unit vectors.

Mean (average) 'circulation' per unit area taken at the point when the elemental area becomes infinitesimally small.

circulation and curl

$$circulation = \oint_C \vec{A}(\vec{r}) \bullet d\vec{l}$$

C



Consider an open surface *S*, bounded by a closed curve *C*.

Circulation depends on the value of the vector at *all the* points on C; it is not a scalar *field even if it is a scalar quantity. It is not a scalar point function*.

$$\lim_{\delta S \to 0} \frac{\oint_C \vec{A}(\vec{r}).d\vec{l}}{\delta S} = \\ = (\vec{\nabla} \times \vec{A}) \bullet \hat{n}$$

Shrink the closed path C; in the limit, the circulation would vanish;
and so would the area S bounded by C.
However, the ratio itself is finite in the limit;
it is a local quantity at that point.

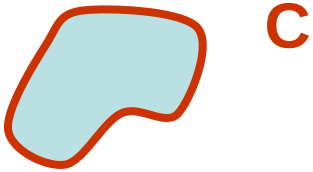
'Limiting' circulation per unit area

circulation and curl

$$circulation = \oint_C \vec{A}(\vec{r}) \bullet d\vec{l}$$

$$\lim_{\delta S \to 0} \frac{\oint_C \vec{A}(\vec{r}).d\vec{l}}{\delta S} = (\vec{\nabla} \times \vec{A}) \bullet \hat{n}$$

'Limiting' circulation per unit area



This limiting ratio defines a component of the curl of the vector field; the curl itself is defined through three orthonormal components in the basis $\{\hat{n}_1, \hat{n}_2, \hat{n}_3\}$

$$\lim_{\delta S \to 0} \frac{\oint_C \vec{A}(\vec{r}).d\vec{l}}{\delta S} = (\vec{\nabla} \times \vec{A}) \bullet \hat{n}$$

Curl measures how much the vector "curls" around at a point

- curl of a vector field at a point represents the net circulation of the field around that point.

- the magnitude of curl at a given point represents the *maximum circulation* at that point.

- the direction of the curl vector is normal to the surface on which the circulation (determined as per the right-hand-rule) is the greatest.

If $\vec{\nabla} \times \vec{F} = \vec{0}$ in a region then there would be no curliness/rotation, and the field is called *irrotational.*

Remember!

$$\lim_{\delta S \to 0} \frac{\oint_C \vec{A}(\vec{r}).d\vec{l}}{\delta S} = (\vec{\nabla} \times \vec{A}) \bullet \hat{n}$$

The criterion that a force field is conservative is that its path integral over a closed loop (i.e. "circulation") is zero. This is equivalent to the condition that $\vec{\nabla} \times \vec{F} = \vec{0}$

If $\vec{\nabla} \times \vec{F} = \vec{0}$ in a region, then there would be no curliness (rotation), and the field is called *irrotational.*

Conservative force fields: IRROTATIONAL

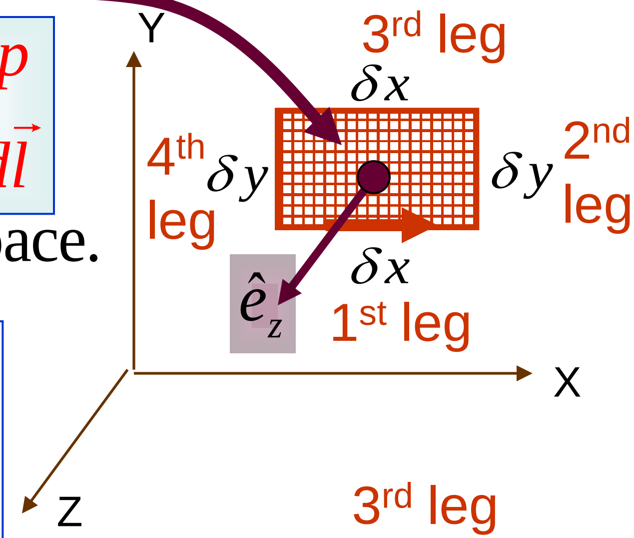
Examples for irrotational fields: electrostatic, gravitational

Curl in Cartesian Co-ordinate

Consider a point $P(x_0, y_0, z_0)$.

add up $\vec{A}(\vec{r}).d\vec{l}$

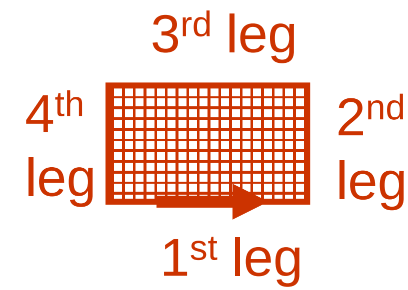
Consider a vector field $\vec{A}(\vec{r})$ in some region of space.



Circulation over the peremeter of an elemental surface to which the normal is $\hat{e}_z$ is

$$\left[A_x\left(x_0, y_0 - \frac{\delta y}{2}, z_0\right) - A_x\left(x_0, y_0 + \frac{\delta y}{2}, z_0\right)\right]\delta x +$$

$$\left[A_y\left(x_0 + \frac{\delta x}{2}, y_0, z_0\right) - A_y\left(x_0 - \frac{\delta x}{2}, y_0, z_0\right)\right]\delta y$$



Closed path C which bounds an elemental surface

$$\oint_C \vec{A}(\vec{r}).d\vec{l} = -\frac{\partial A_x}{\partial y}\delta y \delta x + \frac{\partial A_y}{\partial x}\delta x \delta y$$

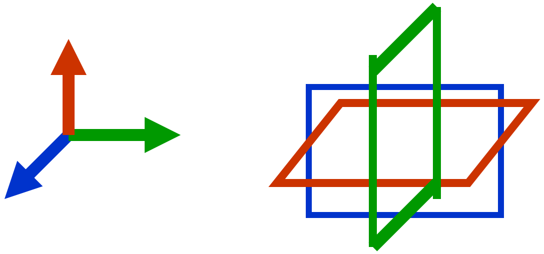
$$\Rightarrow (curl\ \vec{A}) \bullet \hat{e}_z = -\frac{\partial A_x}{\partial y} + \frac{\partial A_y}{\partial x} = (curl \vec{A})_z$$

Make sure that you understand the signs $\pm$

$$\oint_C \vec{A}(\vec{r})\bullet d\vec{l} = -\frac{\partial A_x}{\partial y}\delta y\delta x + \frac{\partial A_y}{\partial x}\delta x\delta y$$

Determining now the net circulation per unit area:

$$(curl\ \vec{A})\bullet \hat{e}_z = -\frac{\partial A_x}{\partial y} + \frac{\partial A_y}{\partial x} = (\vec{\nabla}\times\vec{A})_z$$



Color coded arrows are unit vectors orthogonal to the three mutually orthogonal surface elements bounded by their perimeters.

Similarly if we get circulation per unit area along other two orthogonal closed paths and add up, we get:

$$Curl\,\vec{A} = \hat{e}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{e}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{e}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

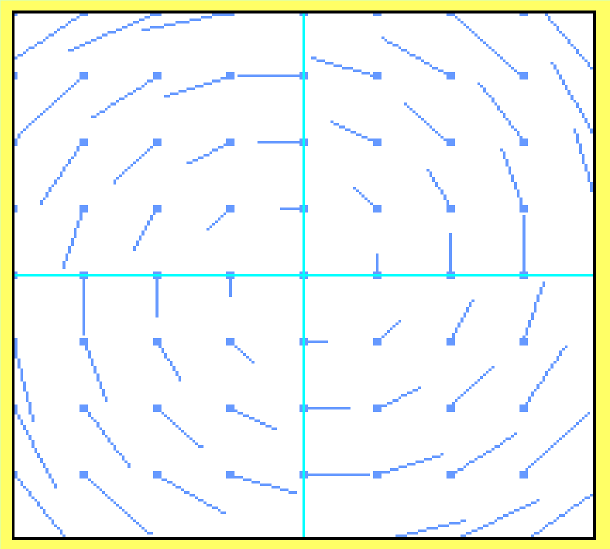
$$curl\,\vec{A} = \hat{e}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{e}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{e}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

The Cartesian expression for curl of a vector field can be expressed as a determinant; but it is, of course, not a determinant!

$$curl\ \vec{A} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

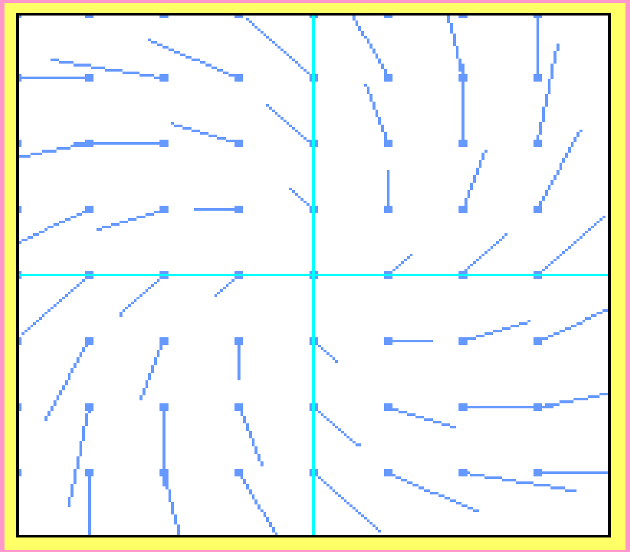
Can you interchange the 2nd and the 3rd row and change the sign of this 'determinant'? The curl is *not* a cross product of two vectors; the gradient is a vector operator!

examples of rotational fields, nonzero curl



$$\vec{V}(x,y) = -y\hat{e}_x + x\hat{e}_y$$

$$\vec{\nabla}\times\vec{V} = 2\hat{e}_z$$

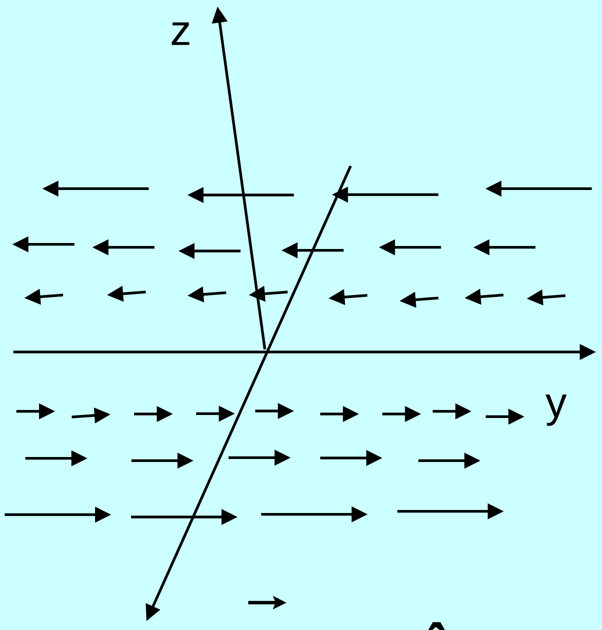


$$\vec{A}(x,y)=(x-y)\hat{e}_x + (x+y)\hat{e}_y$$

$$\vec{\nabla}\times\vec{A} = 2\hat{e}_z$$

curl : along the positive z-axis

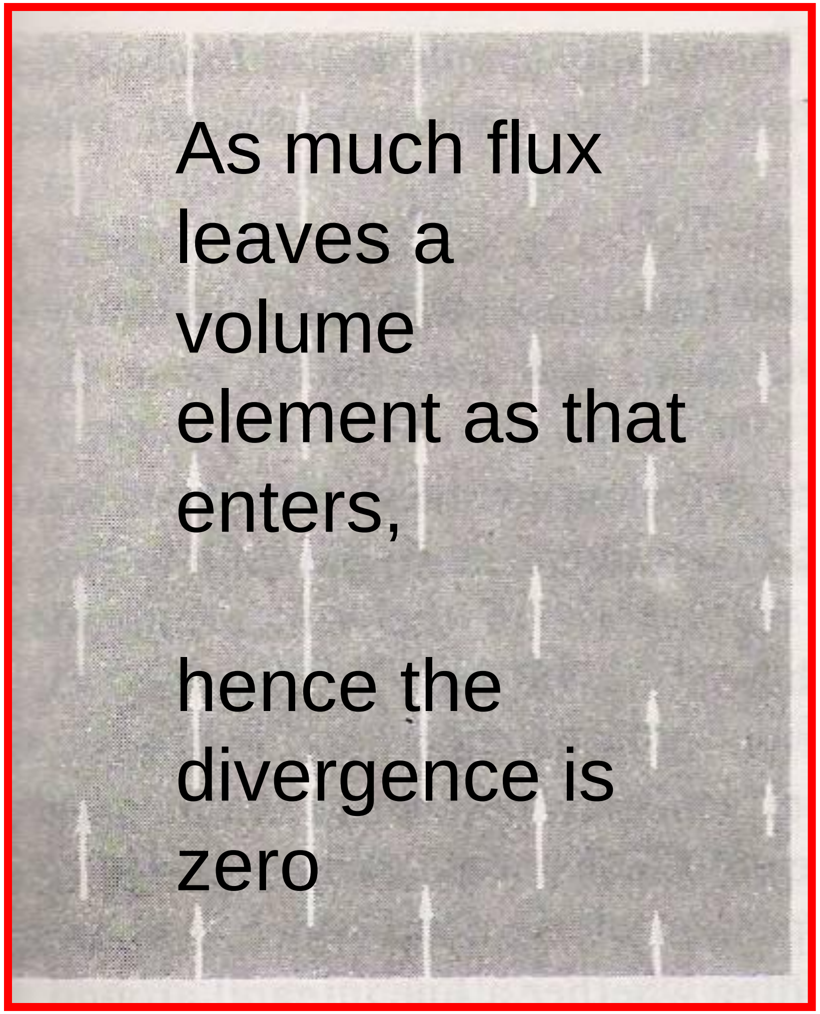
rotational fields, nonzero curl



$$\vec{v} = x\hat{e}_y$$

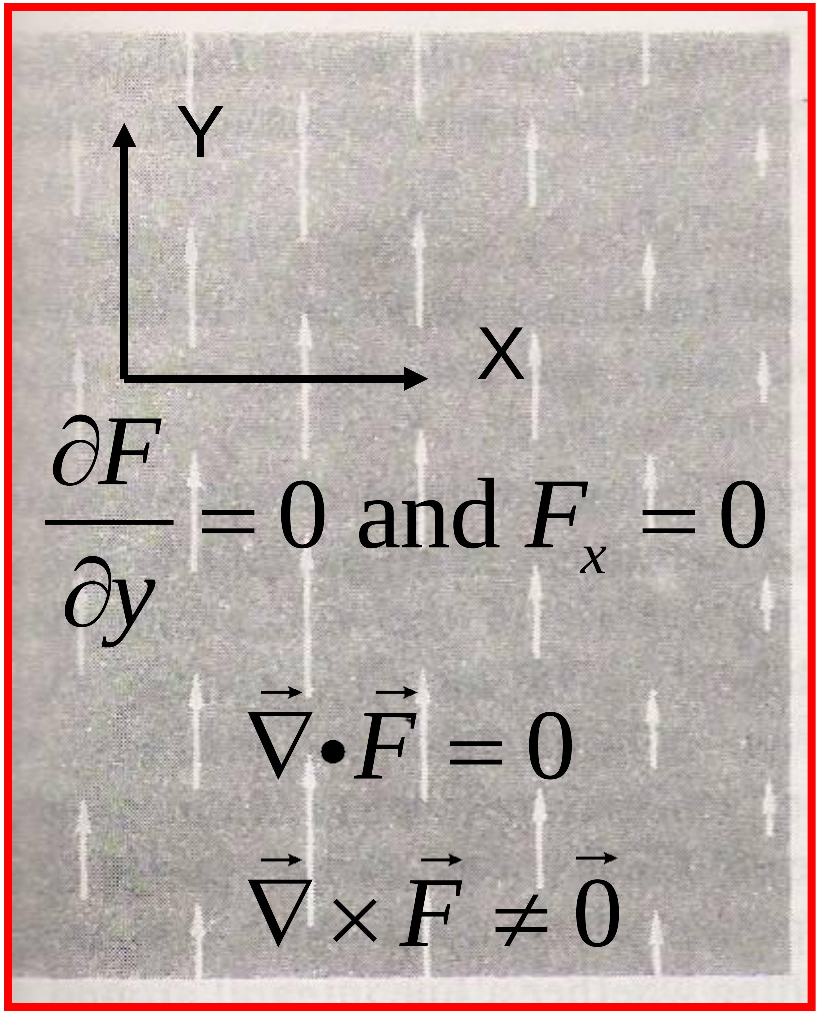
$$\vec{\nabla} \times \vec{v} = \hat{e}_z$$

What is the DIVERGENCE and the CURL of the following vector field?



As much flux leaves a volume element as that enters,

hence the divergence is zero



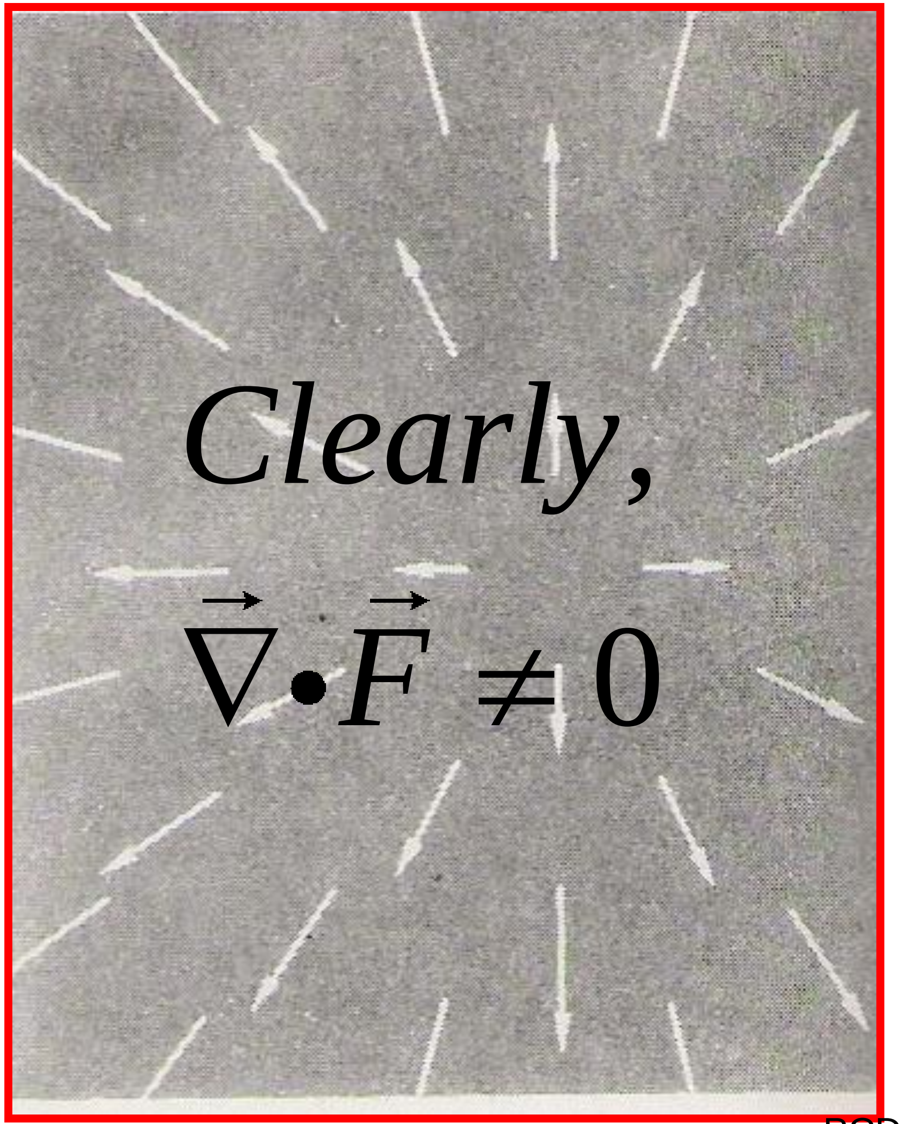
$$\frac{\partial F}{\partial y} = 0 \text{ and } F_x = 0$$

$$\vec{\nabla} \bullet \vec{F} = 0$$

$$\vec{\nabla} \times \vec{F} \neq \vec{0}$$

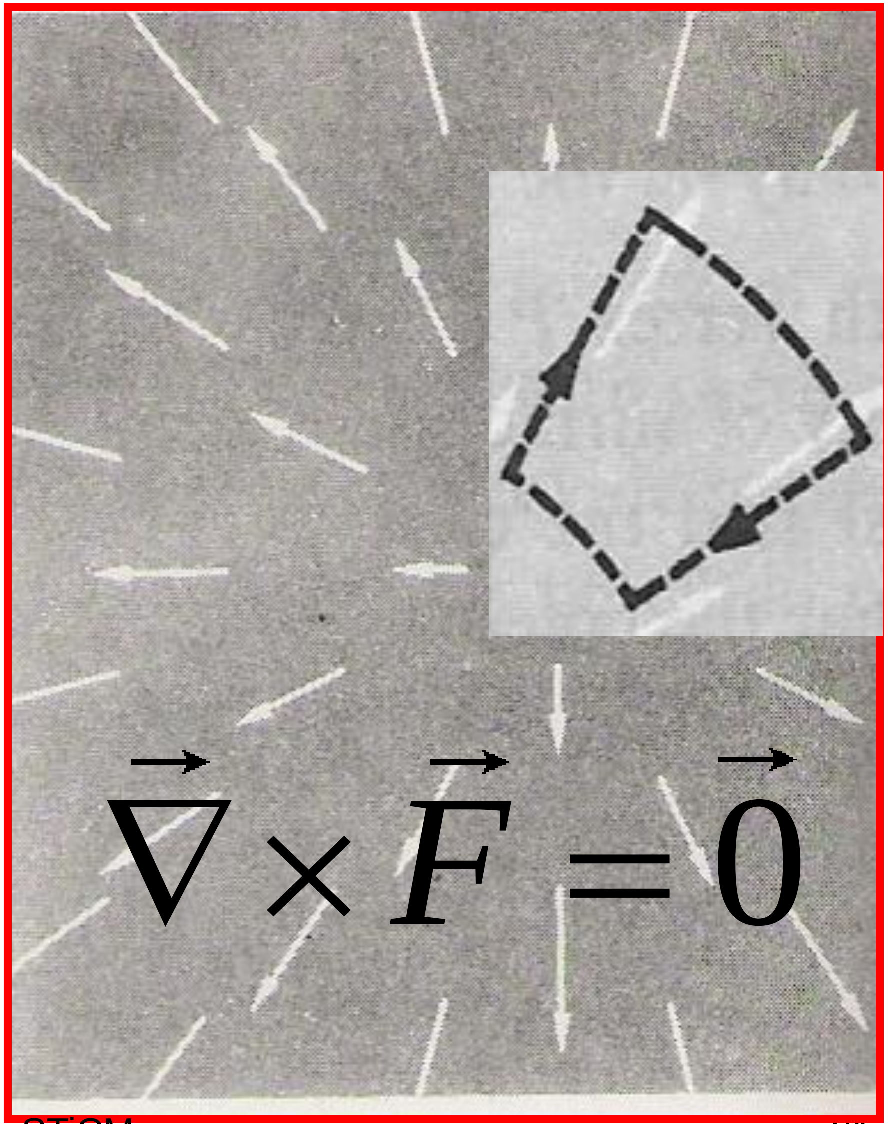
Reference: Berkeley Physics Course, Volume I

What is the DIVERGENCE and the CURL of the following vector field?



Clearly,

$$\vec{\nabla} \bullet \vec{F} \neq 0$$



$$\vec{\nabla} \times \vec{F} = \vec{0}$$

Reference: Berkeley Physics Course, Volume I

curl of a gradient is zero

$$\vec{\nabla}\phi = \hat{e}_x \frac{\partial \phi}{\partial x} + \hat{e}_y \frac{\partial \phi}{\partial y} + \hat{e}_z \frac{\partial \phi}{\partial z}$$

$$\vec{\nabla} \times \vec{\nabla}\phi = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \end{vmatrix}$$

The final result will be independent of the coordinate system.

$$\vec{\nabla} \times \vec{\nabla}\varphi =$$

$$= \hat{e}_x \left(\frac{\partial^2 \varphi}{\partial y \partial z} - \frac{\partial^2 \varphi}{\partial z \partial y} \right) + \hat{e}_y \left(\frac{\partial^2 \varphi}{\partial z \partial x} - \frac{\partial^2 \varphi}{\partial x \partial z} \right) + \hat{e}_z \left(\frac{\partial^2 \varphi}{\partial x \partial y} - \frac{\partial^2 \varphi}{\partial y \partial x} \right)$$

$$= \vec{0}$$

Recall: from Unit 5

Motion in a rotating coordinate system of reference.

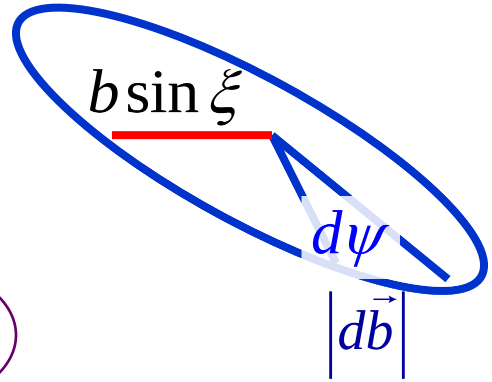
$$d\vec{b} = \vec{b}(t+dt) - \vec{b}(t) = |d\vec{b}|\,\hat{u}$$

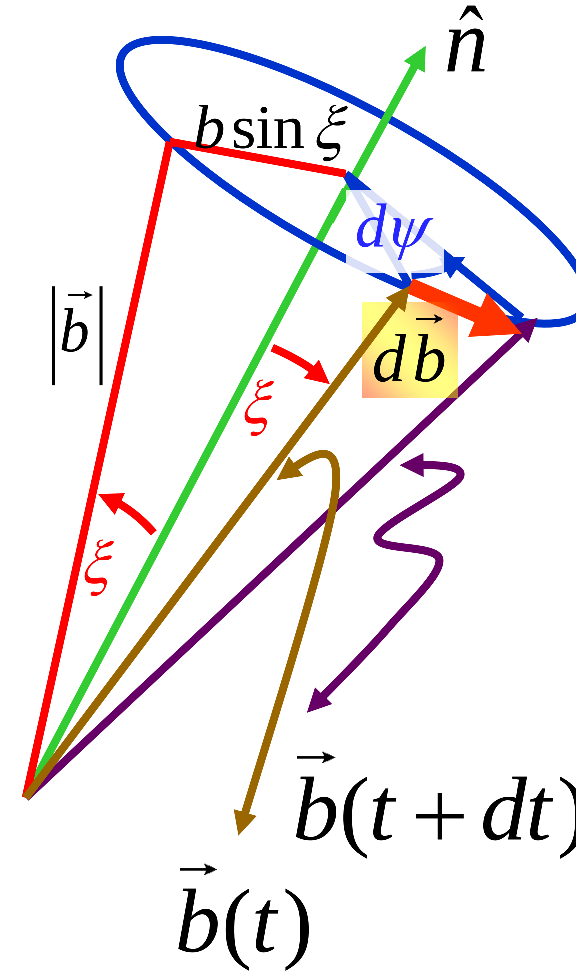
$$\text{where } \hat{u} = \frac{\hat{n}\times\hat{b}}{\left|\hat{n}\times\hat{b}\right|}.$$

$$\xi = \angle(\hat{n},\hat{b})$$

$$\left|d\vec{b}\right| = (b\sin\xi)(d\psi)$$

$$d\vec{b} = (b\sin\xi)(d\psi)\frac{\hat{n}\times\hat{b}}{\left|\hat{n}\times\hat{b}\right|}$$





These two terms are equal and hence cancel.

$$d\vec{b} = d\psi\,\hat{n}\times\vec{b}$$

$$d\vec{b} = (\vec{\omega}dt)\times\vec{b}$$

$$\text{since } \vec{\omega} = \frac{d\psi}{dt}\hat{n}$$

$$\Rightarrow \left(\frac{d}{dt}\right)_I \vec{b} = \vec{\omega}\times\vec{b}$$

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$$\left(\frac{d}{dt}\right)_I \vec{b} = \vec{\omega} \times \vec{b}$$

Remember! The vector $\vec{b}$ itself did not have any time-dependence in the rotating frame.

If $\vec{b}$ has a time dependence in the rotating frame, the following operator equivalence would follow:

$$\left(\frac{d}{dt}\right)_I \vec{b} = \vec{\omega} \times \vec{b} + \left(\frac{d}{dt}\right)_R \vec{b}$$

Operator Equivalence: $$\left(\frac{d}{dt}\right)_I = \left(\frac{d}{dt}\right)_R + \vec{\omega} \times$$

$$\left(\frac{d}{dt}\right)_I = \left(\frac{d}{dt}\right)_R + \vec{\omega}\ \mathrm{x}$$

Recall: from Unit 5

$$\left(\frac{d}{dt}\right)_I \vec{r} = \left(\frac{d}{dt}\right)_R \vec{r} + \vec{\omega}\ \mathrm{x}\ \vec{r}$$

When $\left(\frac{d}{dt}\right)_R \vec{r} = \vec{0},$ $\left(\frac{d}{dt}\right)_I \vec{r} = \vec{\omega}\ \mathrm{x}\ \vec{r}$

$$\left(\frac{d}{dt}\right)_I \vec{r} = \vec{\mathrm{v}}_{\mathrm{I}} = \vec{\omega}\ \mathrm{x}\ \vec{r}$$

$$\left(\frac{d}{dt}\right)_I \vec{r} = \vec{v}_I = \vec{\omega} \text{ x } \vec{r}$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{\nabla} \times \vec{v} = \vec{\nabla} \times (\vec{\omega} \times \vec{r}) = \vec{\nabla} \times \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \omega_x & \omega_y & \omega_z \\ x & y & z \end{vmatrix}$$

$$= \vec{\nabla} \times \left[(\omega_y z - \omega_z y)\hat{e}_x + (\omega_z x - \omega_x z)\hat{e}_y + (\omega_x y - \omega_y x)\hat{e}_z \right]$$

$$= \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (\omega_y z - \omega_z y) & (\omega_z x - \omega_x z) & (\omega_x y - \omega_y x) \end{vmatrix}$$

$$\vec{\nabla} \times \vec{v} = 2(\omega_x \hat{e}_x + \omega_y \hat{e}_y + \omega_z \hat{e}_z) = 2\vec{\omega}$$

The 'curl' of the linear velocity gives a measure of *(twice)* the angular velocity; thus justifying the term 'curl'.

Remember:

$$\hat{u}_i(\vec{r}) \bullet \vec{\nabla} \times \vec{F}(\vec{r}) = \lim_{\Delta S \to 0} \frac{\oint \vec{F}(\vec{r}) \bullet \vec{dr}}{\Delta s}$$

The component of the curl of a vector field in the direction $\hat{u}_i(\vec{r})$ is the circulation about the axis of the vector field per unit area.

It measures the extent to which a particle being carried by the vector field is being rotated about $\hat{u}_i(\vec{r})$.

We shall see in the next class that we are now automatically led to the STOKES THEOREM:

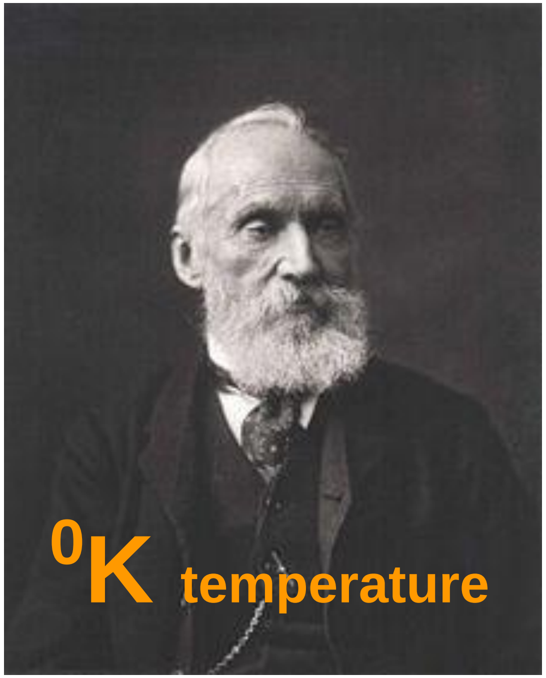
William Thomson, 1st Baron Kelvin (1824-1907)

$$\oint \vec{A}(\vec{r}) \bullet d\vec{l} = \iint (\vec{\nabla} \times \vec{A}) \bullet \overrightarrow{dS}$$

Note!

It is **STOKES'** THEOREM

not STOKE'S THEOREM



George Gabriel Stokes (1819–1903)

This theorem is named after George Gabriel Stokes (1819–1903), although the first known statement of the theorem is by William Thomson (Lord Kelvin) and appears in a letter of his to Stokes in July 1850.

Reference: http://www.123exp-math.com/t/01704066342/

We shall take a break here…….

Questions ? Comments ?

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Next: L30

Unit 9 – Fluid Flow / Bernoulli's principle

….. but *which* Bernoulli ?

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STiCM

Select / Special Topics in Classical Mechanics

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STiCM Lecture 30

Unit 9 : **Fluid Flow, Bernoulli’s Principle**

Curl of a vector, Fluid Mechanics / Electrodynamics, etc.

$$\hat{u}_i(\vec{r}) \bullet \vec{\nabla} \times \vec{F}(\vec{r}) = \lim_{\Delta S \to 0} \frac{\oint \vec{F}(\vec{r}) \bullet \vec{dr}}{\Delta s}$$

The component of the curl of a vector field in the direction $\hat{u}_i(\vec{r})$ is the circulation about the axis of the vector field per unit area.

ABOVE RELATION: provides complete DEFINITION of CURL of a VECTOR.

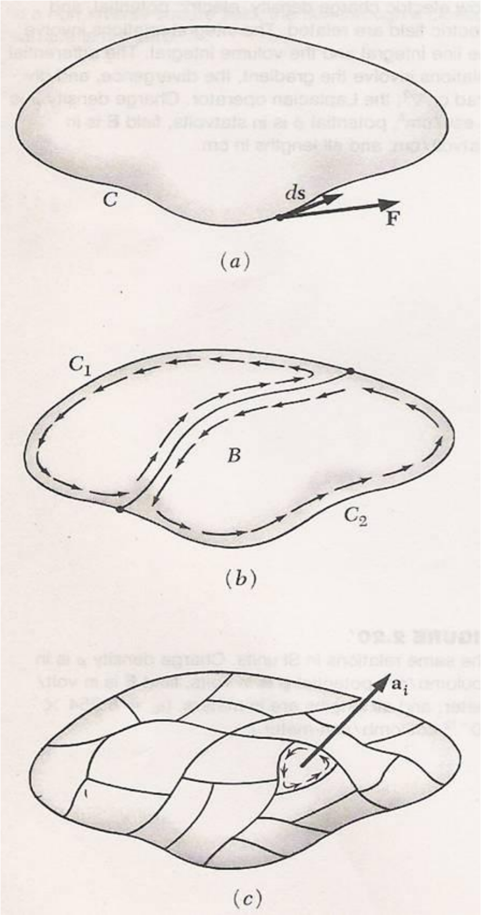
$$\left\{\hat{u}_i(\vec{r});\ i = 1, 2, 3\right\}\ \textit{orthonormal basis}$$

Proof of Stokes' theorem follows from the very definition of the curl:

$$Definition: \quad (curl\ \vec{A}) \bullet \hat{n} = \lim_{\delta S \to 0} \frac{\oint_C \vec{A}(\vec{r}) \bullet d\vec{l}}{\delta S} = (\vec{\nabla} \times \vec{A}) \bullet \hat{n}$$

For a tiny path δC, which binds a tiny area δS,

$$\oint_{\delta C} \vec{A} \bullet d\vec{l} = \delta S \times (curl\ \vec{A}) \bullet \hat{n} = curl\vec{A} \bullet \overrightarrow{\delta S}$$



We can split up a finite area S into infinitesimal bits δS_i bound by tiny curves δC_i

$$\oint_C \vec{A}(\vec{r}) \bullet \overrightarrow{dl} = \sum_{i=1}^{n} \oint_{\delta C_i} \vec{A}(\vec{r}) \bullet \overrightarrow{dl} = \sum_{i=1}^{n} \iint curl\,\vec{A}(\vec{r}) \bullet \overrightarrow{dS}$$

$$\oint_C \vec{A}(\vec{r}) \bullet \overrightarrow{dl} = \iint curl\,\vec{A}(\vec{r}) \bullet \overrightarrow{dS}$$

Stokes' theorem

consider a surface S enclosed by a curve C

Stokes' theorem

$$\oint_C \vec{A}(\vec{r}) \bullet \vec{dl} = \iint curl\,\vec{A}(\vec{r}) \bullet dS\hat{n}$$

The Stokes theorem relates the line integral of a vector about a closed curve to the surface integral of its curl over the enclosed area that the closed curve binds.

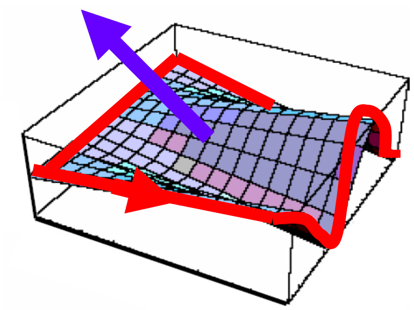
Any surface bound by the closed curve will work; you can pinch the butterfly net and distort the shape of the net any which way – it won't matter!

consider a surface S enclosed by a curve C

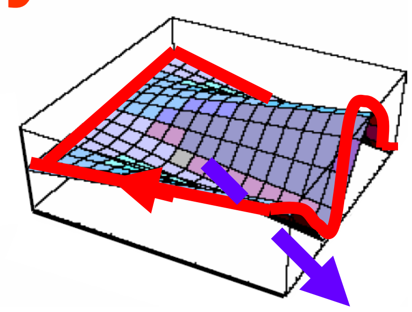
Stokes' theorem

$$\oint_C \vec{A}(\vec{r}) \bullet \vec{dl} = \iint curl\,\vec{A}(\vec{r}) \bullet dS\hat{n}$$

The direction of the vector surface element that appears in the right hand side of the above equation must be defined in a manner that is consistent with the sense in which the closed path integral in the left hand side is evaluated.



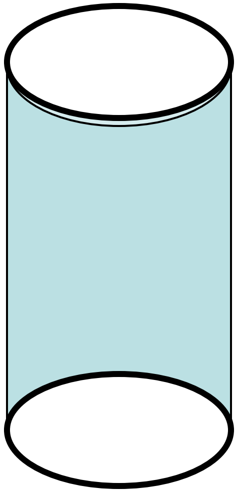
C traversed one way



C traversed the other way

The right-hand-screw convention must be followed.

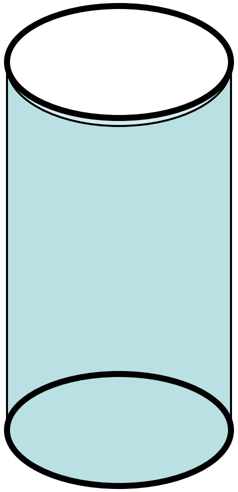
Non-orientable surfaces



The surface under consideration, however, better be a ‘well-behaved’ surface!

A cylinder open at both ends is not a ‘well-behaved’ surface!

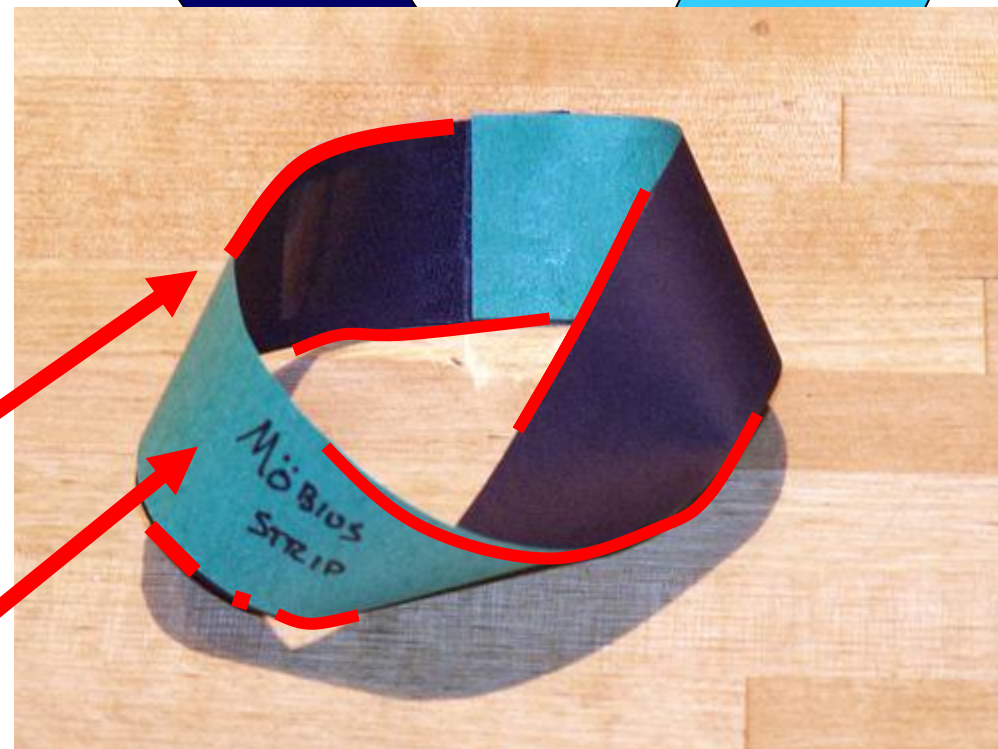
A cylinder open at only one end is ‘well-behaved’; isn’t it already like the butterfly net?



Consider a rectangular strip of paper, spread flat at first, and given two colors on opposite sides.

The surface under consideration, however, better be a ‘well-behaved’ surface!

Now, flip it and paste the short edges on each other as shown.



Is the resulting object three-dimensional?

How many ***‘edges’*** does it have?

How many *‘sides’* does it have?

Expression for 'curl' in cylindrical polar coordinate system $\left\{\hat{e}_\rho, \hat{e}_\varphi, \hat{e}_z\right\}$

$$\vec{\nabla} \times \vec{A} =$$

$$\left[\hat{e}_\rho \frac{\partial}{\partial \rho} + \hat{e}_\varphi \frac{1}{\rho}\frac{\partial}{\partial \varphi} + \hat{e}_z \frac{\partial}{\partial z}\right] \times \left[\hat{e}_\rho A_\rho(\rho,\varphi,z) + \hat{e}_\varphi A_\varphi(\rho,\varphi,z) + \hat{e}_z A_z(\rho,\varphi,z)\right]$$

$$\vec{\nabla} \times \vec{A} = \left[\hat{e}_\rho \frac{\partial}{\partial \rho}\right] \times \left[\hat{e}_\rho A_\rho(\rho,\varphi,z) + \hat{e}_\varphi A_\varphi(\rho,\varphi,z) + \hat{e}_z A_z(\rho,\varphi,z)\right] +$$

$$\left[\hat{e}_\varphi \frac{1}{\rho}\frac{\partial}{\partial \varphi}\right] \times \left[\hat{e}_\rho A_\rho(\rho,\varphi,z) + \hat{e}_\varphi A_\varphi(\rho,\varphi,z) + \hat{e}_z A_z(\rho,\varphi,z)\right] +$$

$$\left[\hat{e}_z \frac{\partial}{\partial z}\right] \times \left[\hat{e}_\rho A_\rho(\rho,\varphi,z) + \hat{e}_\varphi A_\varphi(\rho,\varphi,z) + \hat{e}_z A_z(\rho,\varphi,z)\right]$$

$$\vec{\nabla}\times\vec{A}=$$

$$\left[\hat{e}_\rho\frac{\partial}{\partial\rho}+\hat{e}_\varphi\frac{1}{\rho}\frac{\partial}{\partial\varphi}+\hat{e}_z\frac{\partial}{\partial z}\right]\times\left[\hat{e}_\rho A_\rho(\rho,\varphi,z)+\hat{e}_\varphi A_\varphi(\rho,\varphi,z)+\hat{e}_z A_z(\rho,\varphi,z)\right]$$

$$\left\{\hat{e}_\rho,\hat{e}_\varphi,\hat{e}_z\right\}$$

$$\vec{\nabla}\times\vec{A}=\left[\hat{e}_\rho\frac{\partial}{\partial\rho}\right]\times\left[\hat{e}_\rho A_\rho(\rho,\varphi,z)+\hat{e}_\varphi A_\varphi(\rho,\varphi,z)+\hat{e}_z A_z(\rho,\varphi,z)\right]+$$
$$\left[\hat{e}_\varphi\frac{1}{\rho}\frac{\partial}{\partial\varphi}\right]\times\left[\hat{e}_\rho A_\rho(\rho,\varphi,z)+\hat{e}_\varphi A_\varphi(\rho,\varphi,z)+\hat{e}_z A_z(\rho,\varphi,z)\right]+$$
$$\left[\hat{e}_z\frac{\partial}{\partial z}\right]\times\left[\hat{e}_\rho A_\rho(\rho,\varphi,z)+\hat{e}_\varphi A_\varphi(\rho,\varphi,z)+\hat{e}_z A_z(\rho,\varphi,z)\right]$$

$$\vec{\nabla}\times\vec{A}=\left[\hat{e}_\rho\right]\times\frac{\partial}{\partial\rho}\left[\hat{e}_\rho A_\rho(\rho,\varphi,z)+\hat{e}_\varphi A_\varphi(\rho,\varphi,z)+\hat{e}_z A_z(\rho,\varphi,z)\right]+$$
$$\left[\hat{e}_\varphi\right]\times\left(\frac{1}{\rho}\frac{\partial}{\partial\varphi}\right)\left[\hat{e}_\rho A_\rho(\rho,\varphi,z)+\hat{e}_\varphi A_\varphi(\rho,\varphi,z)+\hat{e}_z A_z(\rho,\varphi,z)\right]+$$
$$\left[\hat{e}_z\right]\times\frac{\partial}{\partial z}\left[\hat{e}_\rho A_\rho(\rho,\varphi,z)+\hat{e}_\varphi A_\varphi(\rho,\varphi,z)+\hat{e}_z A_z(\rho,\varphi,z)\right]$$

Expression for 'curl' in cylindrical polar coordinate system $\left\{\hat{e}_\rho, \hat{e}_\varphi, \hat{e}_z\right\}$

$$\vec{\nabla}\times\vec{A} = \left[\hat{e}_\rho\right]\times\frac{\partial}{\partial\rho}\left[\hat{e}_\rho A_\rho(\rho,\varphi,z)+\hat{e}_\varphi A_\varphi(\rho,\varphi,z)+\hat{e}_z A_z(\rho,\varphi,z)\right]+$$

$$\left[\hat{e}_\varphi\right]\times\left(\frac{1}{\rho}\frac{\partial}{\partial\varphi}\right)\left[\hat{e}_\rho A_\rho(\rho,\varphi,z)+\hat{e}_\varphi A_\varphi(\rho,\varphi,z)+\hat{e}_z A_z(\rho,\varphi,z)\right]+$$

$$\left[\hat{e}_z\right]\times\frac{\partial}{\partial z}\left[\hat{e}_\rho A_\rho(\rho,\varphi,z)+\hat{e}_\varphi A_\varphi(\rho,\varphi,z)+\hat{e}_z A_z(\rho,\varphi,z)\right]$$

$$=\hat{e}_\rho\left(\frac{1}{\rho}\frac{\partial A_z}{\partial\varphi}-\frac{\partial A_\varphi}{\partial z}\right)+\hat{e}_\varphi\left(\frac{\partial A_\rho}{\partial z}-\frac{\partial A_z}{\partial\rho}\right)+\hat{e}_z\frac{1}{\rho}\left[\frac{\partial\left(\rho A_\varphi\right)}{\partial\rho}-\frac{\partial A_\rho}{\partial\varphi}\right]$$

Expression for 'curl' in spherical polar coordinate system

$$\left\{\hat{e}_r, \hat{e}_\theta, \hat{e}_\varphi\right\}$$

$$\vec{\nabla}\times\vec{A}=$$

$$=\left(\hat{e}_r\frac{\partial}{\partial r}+\hat{e}_\theta\frac{1}{r}\frac{\partial}{\partial\theta}+\hat{e}_\varphi\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\right)\times\left(\hat{e}_r A_r(r,\theta,\varphi)+\hat{e}_\theta A_\theta(r,\theta,\varphi)+\hat{e}_\varphi A_\varphi(r,\theta,\varphi)\right)$$

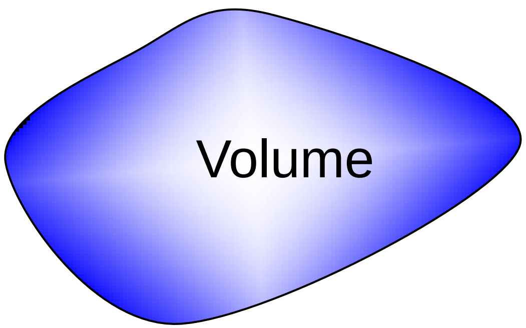
$$\vec{\nabla}\times\vec{A}=\hat{e}_r\frac{1}{r\sin\theta}\left\{\frac{\partial}{\partial\theta}\left(\sin\theta A_\varphi\right)-\frac{\partial A_\theta}{\partial\varphi}\right\}$$

$$+\hat{e}_\theta\frac{1}{r}\left\{\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi}-\frac{\partial}{\partial r}\left(rA_\varphi\right)\right\}$$

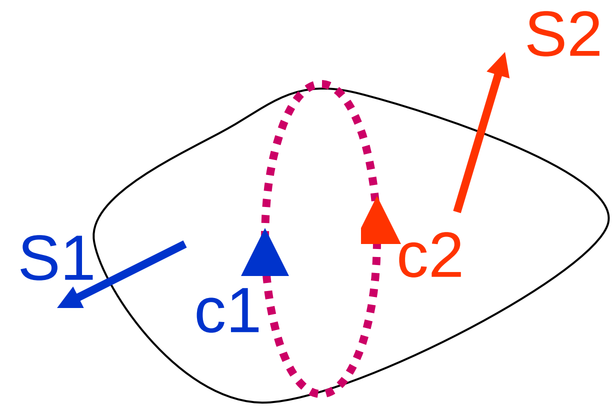
$$+\hat{e}_\varphi\frac{1}{r}\left\{\frac{\partial}{\partial r}\left(rA_\theta\right)-\frac{\partial A_r}{\partial\theta}\right\}$$

An important identity: divergence of a curl is zero

Gauss' divergence theorem $\iiint_V \vec{\nabla}\cdot(\vec{\nabla}\times\vec{A})\,d\tau = \oiint_S \left(\vec{\nabla}\times\vec{A}\right)\bullet\overrightarrow{dS}$



Surface enclosing a volume



Applying Stoke's theorem

$$\oiint_S (\vec{\nabla}\times\vec{A})\cdot d\vec{S} = \iint_{S_1} (\vec{\nabla}\times\vec{A})\cdot\hat{n}_1 dS_1 + \iint_{S_2} (\vec{\nabla}\times\vec{A})\cdot\hat{n}_2 dS_2$$

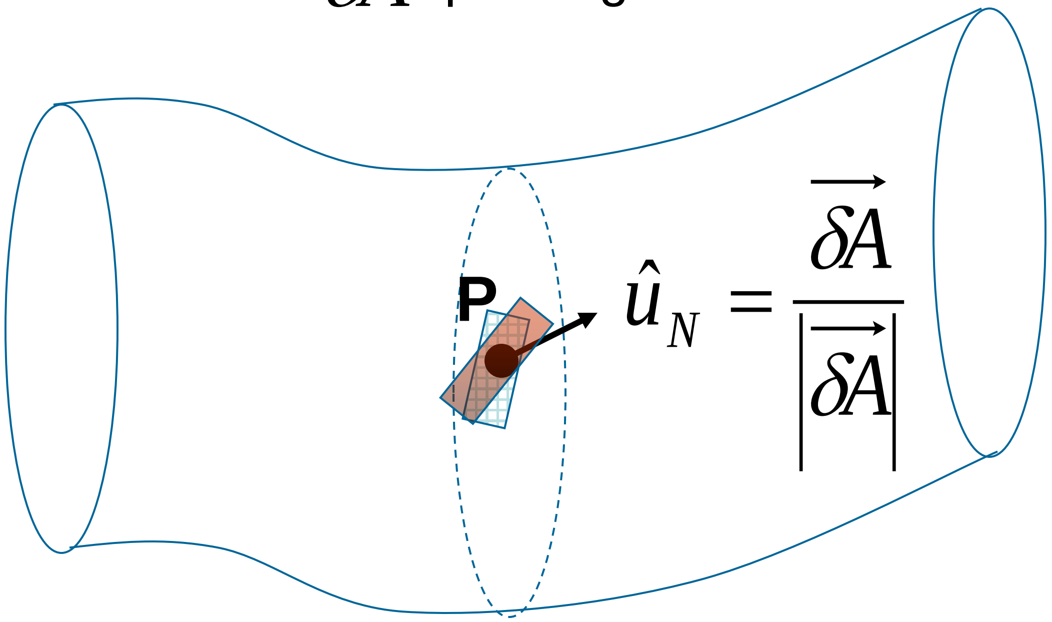
$$= \oint_{C_1} \vec{A}\cdot d\vec{l} + \oint_{C_2} \vec{A}\cdot d\vec{l} = 0$$

$$\boxed{\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0}$$

To understand the term 'ideal' fluid, we first define (i) 'tension', (ii) 'compressions' and (iii) 'shear'.

Consider the force $\vec{F}$ on a tiny elemental area $\overrightarrow{\delta A}$ passing through point P in the liquid.

Stress at the point P is $\vec{S}$.



$\vec{S} \bullet \hat{u}_N = \left|\vec{S}\right|$ → $\vec{S}$: **Tension**

$\vec{S} \bullet \hat{u}_N = 0$ → $\vec{S}$: **Shear**

$\vec{S} \bullet \hat{u}_N = -\left|\vec{S}\right|$ → $\vec{S}$: **Compression**

The unit normal $\hat{u}_N$ can take any orientation.

An ideal fluid is one in which stress at any point is essentially one of COMPRESSION.

The curl of a vector is an important quantity.

A very important theorem in vector calculus is the Helmholtz theorem which states that given the divergence and the curl of a vector field, and appropriate boundary conditions, the vector field is completely specified. You will use this to study Maxwell's equations which provide the curl and the divergence of the electromagnetic field.

Besides, the 'curl' finds direct application also in the derivation of the Bernoulli's principle, as shown below.

2 sons of Nicolaus Bernoulli's

Bernoulli Family Math/Phys Tree

Bernoulli brothers


Jacob I
1654-1705

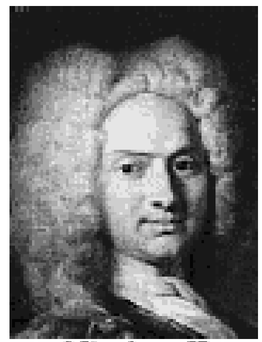

Johann I
1667-1748

Posed the brachistrocrone problem

Bernoulli's Principle


Jacob HERMANN
1678-1733

Nicolaus I
1687-1759


Nicolaus II
1695-1726


Daniel
1700-1782


Johann II
1710-1790

Johann's work was assembled by the Marquis de l;Hospital (1661-1704) under a strange financial agreement with Johann in 1696 into the first calculus textbook. The famous method of evaluating the indeterminate form 0/0 got to be known as l'Hospital's rule.


Johann III
1744-1807


Jacob II
1759-1789

Reference: http://www.york.ac.uk/depts/maths/histstat/people/bernoulli_tree.htm

References to read more about the Bernoulli Family:

http://www.york.ac.uk/depts/maths/histstat/people/bernoulli_tree.htm

http://library.thinkquest.org/22584/temh3007.htm

"...it would be better for the true physics if there were no mathematicians on earth".

Quoted in *The Mathematical Intelligencer* **13** (1991).

http://www-groups.dcs.st-and.ac.uk/~history/Quotations/Bernoulli_Daniel.html

Daniel Bernoulli
1700 - 1782

http://www-groups.dcs.st-and.ac.uk/~history/PictDisplay/Bernoulli_Daniel.html

$$\frac{d\vec{v}}{dt} = \left[\frac{d}{dt}\right]\vec{v}\left(\vec{r}(t), t\right) = \left[\frac{d}{dt}\right]\vec{v}\left(x(t), y(t), z(t), t\right)$$

$$= \frac{\partial\vec{v}}{\partial x}\frac{dx}{dt} + \frac{\partial\vec{v}}{\partial y}\frac{dy}{dt} + \frac{\partial\vec{v}}{\partial z}\frac{dz}{dt} + \frac{\partial\vec{v}}{\partial t}$$

$$\frac{d\vec{v}}{dt} = \frac{dx}{dt}\frac{\partial\vec{v}}{\partial x} + \frac{dy}{dt}\frac{\partial\vec{v}}{\partial y} + \frac{dz}{dt}\frac{\partial\vec{v}}{\partial z} + \frac{\partial\vec{v}}{\partial t}$$

$$= \left[\vec{v}\bullet\vec{\nabla} + \frac{\partial}{\partial t}\right]\vec{v}$$

"CONVECTIVE DERIVATIVE OPERATOR"

$$i.e. \quad \frac{d}{dt} \equiv \left[\vec{v}\bullet\vec{\nabla} + \frac{\partial}{\partial t}\right]$$

The term 'convection' is a reminder of the fact that in the convection process, the transport of a material particle is involved.

Result of the previous unit, Unit 8:

$$\left[\vec{\mathrm{v}}\bullet\vec{\nabla}+\frac{\partial}{\partial t}\right]\vec{\mathrm{v}}(\vec{r},t)=\frac{d}{dt}\vec{\mathrm{v}}(\vec{r},t)=\frac{-\vec{\nabla}p}{\rho(\vec{r})}-\vec{\nabla}\varphi$$

Use now the following vector identity:

$$\vec{\nabla}\left(\vec{A}\bullet\vec{B}\right)=$$

$$\left(\vec{A}\bullet\vec{\nabla}\right)\vec{B}+\left(\vec{B}\bullet\vec{\nabla}\right)\vec{A}+\vec{A}\times\left(\vec{\nabla}\times\vec{B}\right)+\vec{B}\times\left(\vec{\nabla}\times\vec{A}\right)$$

$$\vec{\nabla}\left(\vec{\mathrm{v}}\bullet\vec{\mathrm{v}}\right)=$$

$$\left(\vec{\mathrm{v}}\bullet\vec{\nabla}\right)\vec{\mathrm{v}}+\left(\vec{\mathrm{v}}\bullet\vec{\nabla}\right)\vec{\mathrm{v}}+\vec{\mathrm{v}}\times\left(\vec{\nabla}\times\vec{\mathrm{v}}\right)+\vec{\mathrm{v}}\times\left(\vec{\nabla}\times\vec{\mathrm{v}}\right)$$

$$i.e.\ \frac{1}{2}\vec{\nabla}\left(\vec{\mathrm{v}}\bullet\vec{\mathrm{v}}\right)=\left(\vec{\mathrm{v}}\bullet\vec{\nabla}\right)\vec{\mathrm{v}}+\vec{\mathrm{v}}\times\left(\vec{\nabla}\times\vec{\mathrm{v}}\right)$$

$$\frac{1}{2}\vec{\nabla}\left(\vec{\mathrm{v}}\bullet\vec{\mathrm{v}}\right)-\vec{\mathrm{v}}\times\left(\vec{\nabla}\times\vec{\mathrm{v}}\right)+\frac{\partial\vec{\mathrm{v}}}{\partial t}=\frac{-\vec{\nabla}p}{\rho(\vec{r})}-\vec{\nabla}\phi$$

$$\frac{1}{2}\vec{\nabla}\left(\vec{v}\bullet\vec{v}\right)-\vec{v}\times\left(\vec{\nabla}\times\vec{v}\right)+\frac{\partial\vec{v}}{\partial t}=\frac{-\vec{\nabla}p}{\rho(\vec{r})}-\vec{\nabla}\phi$$

$$Now,\ \frac{\vec{\nabla}p(\vec{r})}{\rho(\vec{r})}\approx\vec{\nabla}\left\{\frac{p(\vec{r})}{\rho}\right\}$$

$$\frac{1}{2}\vec{\nabla}\left(\vec{v}\bullet\vec{v}\right)-\vec{v}\times\left(\vec{\nabla}\times\vec{v}\right)+\frac{\partial\vec{v}}{\partial t}=-\vec{\nabla}\left\{\frac{p(\vec{r})}{\rho}\right\}-\vec{\nabla}\phi$$

i.e.,

$$\frac{1}{2}\vec{\nabla}\left(\vec{v}\bullet\vec{v}\right)-\vec{v}\times\left(\vec{\nabla}\times\vec{v}\right)+\frac{\partial\vec{v}}{\partial t}=-\vec{\nabla}\left\{\frac{p(\vec{r})}{\rho}+\phi\right\}$$

$$\frac{1}{2}\vec{\nabla}\left(\vec{v}\bullet\vec{v}\right)-\vec{v}\times\left(\vec{\nabla}\times\vec{v}\right)+\frac{\partial\vec{v}}{\partial t}=-\vec{\nabla}\left\{\frac{p(\vec{r})}{\rho}+\phi\right\}$$

Recall that : $\left(\frac{d}{dt}\right)_I \vec{r} = \left(\frac{d}{dt}\right)_R \vec{r} \;+\; \vec{\omega}\ \text{x}\ \vec{r}$

$\vec{v}_I = \vec{v}_R \;+\; \vec{\omega}\ \text{x}\ \vec{r}_R$, where $\vec{v}_I$ is just the velocity that is employed in the equation of motion for the fluid.

$$\therefore\ \vec{\nabla}\times\vec{v}_I = \vec{\nabla}\times\vec{v}_R \;+\; \vec{\nabla}\times\left\{\vec{\omega}\ \text{x}\ \vec{r}_R\right\}$$

To determine $\vec{\nabla}\times\left\{\vec{\omega}\ \text{x}\ \vec{r}_R\right\}$ we now use another vector Identity, for the curl of cross-product of two vectors:

$$\vec{\nabla}\times\left(\vec{A}\times\vec{B}\right)=\left(\vec{B}\bullet\vec{\nabla}\right)\vec{A}-\left(\vec{A}\bullet\vec{\nabla}\right)\vec{B}+\vec{A}(\vec{\nabla}\bullet\vec{B})-\vec{B}(\vec{\nabla}\bullet\vec{A})$$

$$\vec{\nabla} \times \vec{v}_I = \vec{\nabla} \times \vec{v}_R + \vec{\nabla} \times \left\{ \vec{\omega} \text{ x } \vec{r}_R \right\}$$

$$\vec{\nabla} \times \left(\vec{A} \times \vec{B} \right) = \left(\vec{B} \bullet \vec{\nabla} \right) \vec{A} - \left(\vec{A} \bullet \vec{\nabla} \right) \vec{B} + \vec{A}(\vec{\nabla} \bullet \vec{B}) - \vec{B}(\vec{\nabla} \bullet \vec{A})$$

$$\vec{\nabla} \times \left(\vec{\omega} \times \vec{r}_R \right) = \left(\vec{r}_R \bullet \vec{\nabla} \right) \vec{\omega} - \left(\vec{\omega} \bullet \vec{\nabla} \right) \vec{r}_R + \vec{\omega}(\vec{\nabla} \bullet \vec{r}_R) - \vec{r}_R(\vec{\nabla} \bullet \vec{\omega})$$

$$\vec{\nabla} \times \left(\vec{\omega} \times \vec{r}_R \right) = -\left(\vec{\omega} \bullet \vec{\nabla} \right) \vec{r}_R + \vec{\omega}(\vec{\nabla} \bullet \vec{r}_R) = 2\vec{\omega}$$

$$\vec{\nabla} \times \vec{v}_I = \vec{\nabla} \times \vec{v}_R + 2\vec{\omega}$$

In the rotating frame, $\vec{v}_R = \vec{0}$,

hence, the VORTICITY, $\vec{\nabla} \times \vec{v}_I = \vec{\chi} = 2\vec{\omega}$

$$\frac{1}{2}\vec{\nabla}\left(\vec{v}\bullet\vec{v}\right)-\vec{v}\times\left(\vec{\nabla}\times\vec{v}\right)+\frac{\partial\vec{v}}{\partial t}=-\vec{\nabla}\left\{\frac{p(\vec{r})}{\rho}+\phi\right\}$$

$$\frac{1}{2}\vec{\nabla}\left|\vec{v}\right|^2-\vec{v}\times\vec{\chi}+\frac{\partial\vec{v}}{\partial t}=-\vec{\nabla}\left\{\frac{p(\vec{r})}{\rho}+\phi\right\}$$

$$\frac{\partial\vec{v}}{\partial t}-\vec{v}\times\vec{\chi}=-\vec{\nabla}\left\{\frac{p(\vec{r})}{\rho}+\phi\right\}-\frac{1}{2}\vec{\nabla}\left|\vec{v}\right|^2$$

$$\frac{\partial\vec{v}}{\partial t}-\vec{v}\times\vec{\chi}=-\vec{\nabla}\left\{\frac{p(\vec{r})}{\rho}+\phi+\frac{\left|\vec{v}\right|^2}{2}\right\}$$

For 'STEADY STATE'

$$\frac{\partial\vec{v}}{\partial t}=\vec{0}$$

$$\not{-}\,\vec{v}\times\vec{\chi}=\not{-}\,\vec{\nabla}\left\{\frac{p(\vec{r})}{\rho}+\phi+\frac{\left|\vec{v}\right|^2}{2}\right\}$$

$$\mathit{Hence},\ 0=\vec{v}\bullet\vec{\nabla}\left\{\frac{p(\vec{r})}{\rho}+\phi+\frac{\left|\vec{v}\right|^2}{2}\right\}$$

$$0 = \vec{\mathrm{v}} \bullet \vec{\nabla}\left\{\frac{p(\vec{r})}{\rho} + \phi + \frac{\left|\vec{\mathrm{v}}\right|^2}{2}\right\}$$

$$\Rightarrow \vec{\nabla}\left\{\frac{p(\vec{r})}{\rho} + \phi + \frac{\left|\vec{\mathrm{v}}\right|^2}{2}\right\} \text{must be ORTHOGONAL to } \vec{\mathrm{v}},$$

$$i.e., \vec{\nabla}\left\{\frac{p(\vec{r})}{\rho} + \phi + \frac{\left|\vec{\mathrm{v}}\right|^2}{2}\right\} \text{must be ORTHOGONAL to } \mathit{STREAMLINES}$$

$$\Rightarrow \vec{\nabla}\Psi \text{ must be ORTHOGONAL to } \mathit{streamlines},$$

$$\mathit{where}\ \ \Psi = \frac{p(\vec{r})}{\rho} + \phi + \frac{\left|\vec{\mathrm{v}}\right|^2}{2}.$$

$$\Rightarrow \Psi = \frac{p(\vec{r})}{\rho} + \varphi + \frac{\left|\vec{\mathrm{v}}\right|^2}{2} = \text{constant along a given streamline}$$

$$\Rightarrow \Psi = \frac{p(\vec{r})}{\rho} + \phi + \frac{\left|\vec{v}\right|^2}{2} = \text{constant for a given streamline}$$

We derived the above result for a 'STEADY STATE' and made use of the relation

$$-\vec{v} \times \vec{\chi} = -\vec{\nabla}\left\{\frac{p(\vec{r})}{\rho} + \phi + \frac{\left|\vec{v}\right|^2}{2}\right\}$$

$$\frac{\partial \vec{v}}{\partial t} = \vec{0}$$

If the fluid flow is both 'steady state' and 'irrotational',

$$\vec{\nabla} \times \vec{v} = \vec{\chi} = \vec{0}$$

$$\Rightarrow \Psi = \frac{p(\vec{r})}{\rho} + \phi + \frac{\left|\vec{v}\right|^2}{2}$$

Daniel Bernoulli's Theorem

is constant for the entire velocity field in the liquid.

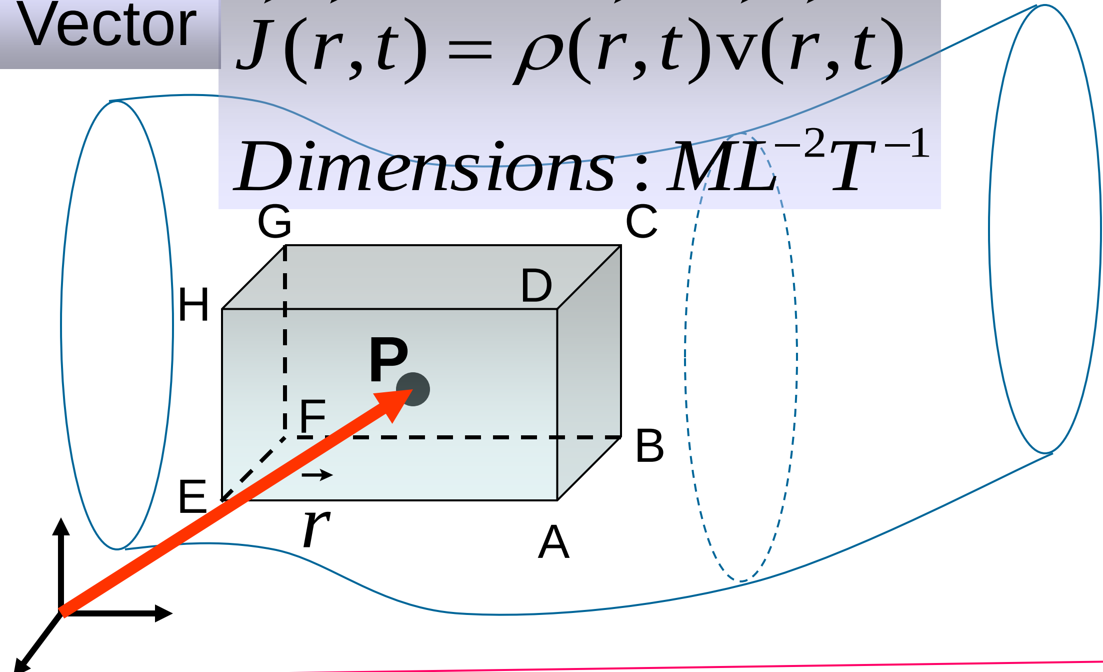
WORK – ENERGY Theorem

Conservation of Energy

Mass Current Density Vector

$$\vec{J}(\vec{r},t) = \rho(\vec{r},t)\vec{v}(\vec{r},t)$$

$$Dimensions: ML^{-2}T^{-1}$$



For Steady State Flow,

$$\rho v A = \text{constant},$$

A : cross-sectional area

since,

$$\underset{\text{volume region}}{\iiint} d\tau \left\{\vec{\nabla} \bullet \vec{J}(\vec{r})\right\} = \underset{\text{surface enclosing that region}}{\iint} \vec{J}(\vec{r}) \bullet \overrightarrow{dS} = 0, \text{ for STEADY STATE as } \frac{\partial \rho}{\partial t} = 0$$

Work done on the fluid by the pressure that the fluid exerts on Face 1 is:

$$\delta W_1 = F_1 \delta s = p_1 A_1 \delta s = p_1 A_1 v_1 \delta t$$

Work done by the fluid on Face 2 is:

$$\delta W_2 = F_2 \delta s = p_2 A_2 \delta s = p_2 A_2 v_2 \delta t$$

Net work done on the fluid in the parallelepiped by the pressure that the fluid exerts on Faces 1 & 2 is:

$$\delta W_1 - \delta W_2 = p_1 A_1 v_1 \delta t - p_2 A_2 v_2 \delta t$$

Net work done on the fluid in the parallelepiped by the pressure that the fluid exerts at Faces 1 & 2 :

$$\delta W_1 - \delta W_2 = p_1 A_1 v_1 \delta t - p_2 A_2 v_2 \delta t$$

Energy gained per unit mass by the fluid as it traverses the x-axis of the parallelepiped across the Faces 1 & 2 :

$$E_2 - E_1 = \frac{\left[p_1 A_1 v_1 - p_2 A_2 v_2\right]\delta t}{\delta m}$$

$$\frac{\left[p_1 A_1 v_1 - p_2 A_2 v_2\right]\delta t}{\delta m} = E_2 - E_1$$

$$= \left[\frac{1}{2}v^2 + \varphi + U_{\text{internal}}\right]_2 - \left[\frac{1}{2}v^2 + \varphi + U_{\text{internal}}\right]_1$$

$$\frac{\left[p_1 A_1 v_1 - p_2 A_2 v_2\right]\delta t}{\rho\left(\delta s A\right)} = \left[\frac{1}{2}v^2 + \varphi + U_{\text{internal}}\right]_2 - \left[\frac{1}{2}v^2 + \varphi + U_{\text{internal}}\right]_1$$

$$\frac{\left[p_1 A_1 v_1 - p_2 A_2 v_2\right]\delta t}{\rho(\delta s A)} = \left[\frac{1}{2}v^2 + \varphi + U_{\text{internal}}\right]_2 - \left[\frac{1}{2}v^2 + \varphi + U_{\text{internal}}\right]_1$$

$$\left[\frac{p_1 A_1 v_1}{\rho(v_1 \delta t) A_1} - \frac{p_2 A_2 v_2}{\rho(v_2 \delta t) A_2}\right]\delta t = \left[\frac{1}{2}v^2 + \varphi + U_{\text{internal}}\right]_2 - \left[\frac{1}{2}v^2 + \varphi + U_{\text{internal}}\right]_1$$

$$\left[\frac{p_1}{\rho} - \frac{p_2}{\rho}\right] = \left[\frac{1}{2}v^2 + \varphi + U_{\text{internal}}\right]_2 - \left[\frac{1}{2}v^2 + \varphi + U_{\text{internal}}\right]_1$$

$$0 = \left[\frac{1}{2}v^2 + \varphi + U_{\text{internal}} + \frac{p}{\rho}\right]_2 - \left[\frac{1}{2}v^2 + \varphi + U_{\text{internal}} + \frac{p}{\rho}\right]_1$$

i.e. $$\frac{1}{2}v^2 + \varphi + U_{\text{internal}} + \frac{p}{\rho} = \text{constant}$$

Daniel Bernoulli's Theorem

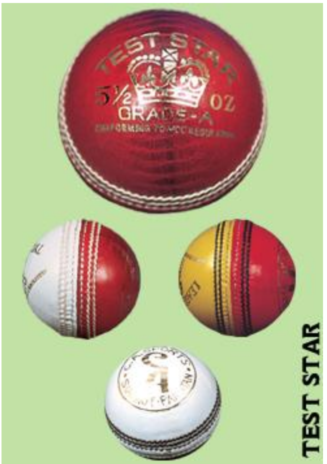
From slide 53:

$$\Psi = \frac{p(\vec{r})}{\rho} + \varphi + \frac{\left|\vec{v}\right|^2}{2}$$

is constant for the entire velocity field in the liquid.

$$\frac{p(\vec{r})}{\rho} + \phi + \frac{|\vec{v}|^2}{2} = \text{constant}$$

The swing of a ball is governed by Bernoulli's theorem.



A swing bowler rubs only one side of the ball. The ball is then more rough on one side than on the other.

Ishant Sharma Inswing / Outswing bowler

A white ball has a thin lacquer that is applied to its surface to avoid discoloring the ball. During play, the shiny surface of the white ball remains shinier than that of a red ball, which has a rougher surface to begin with.

The difference between the rough and shiny surface of a white ball is much more, and thus it swings more than the red ball.

"***Aerodynamics of sports balls***", Rabindra D. Mehta, in Annual Reviews of Fluid Mechanics, 1985. 17: pp. 151-189.

"***On the swing of a cricket ball in flight***", N. G. Barton, *Proc. R. Soc. of London. Ser.* A, 1982. 379 pp. 109-31.

"***An experimental study of cricket ball swing***", Bentley, K., Varty, P., Proudlove, M., Mehta, R. D., Aero. Tech. Note 82-106, Imperial College, London, England, 1982.

"***The effect of humidity on the swing of cricket balls***", Binnie, A. M., International Journal of Mechanical Sciences. 18: pp.497-499, 1976.

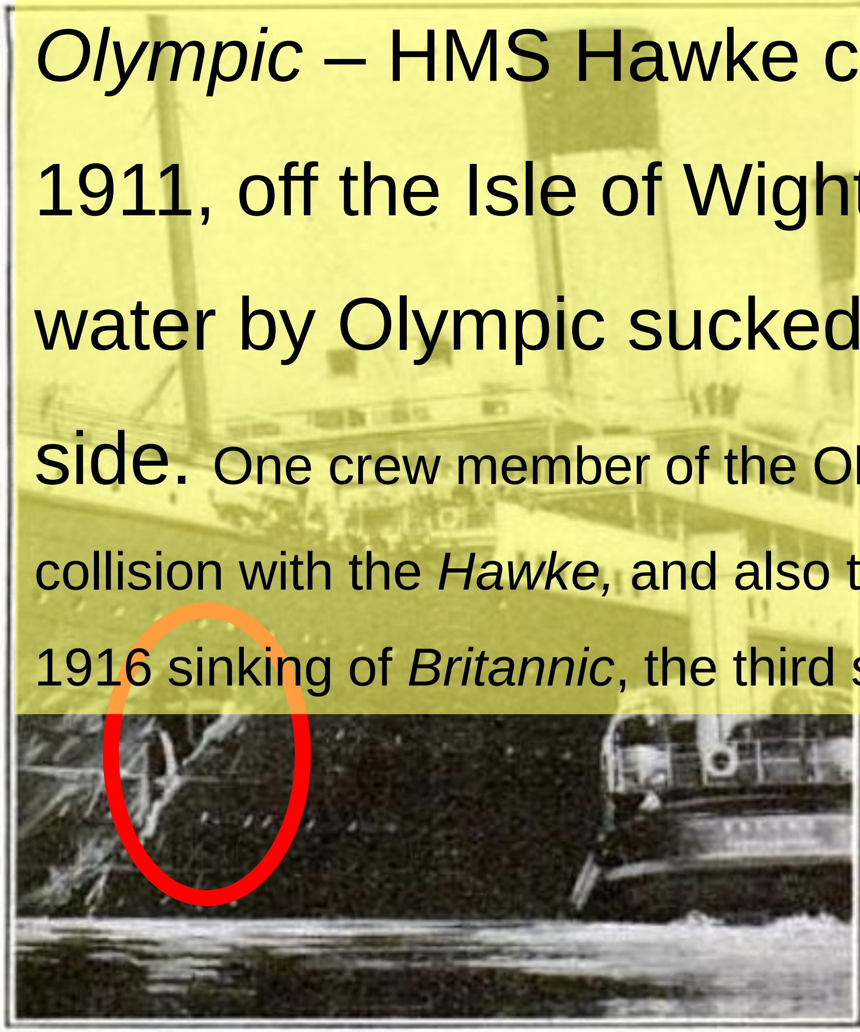
"***The swing of a cricket ball***", Horlock, J. H., in "Mechanics and sport", ed. J. L. Bleustein, pp. 293-303. New York, ASME 1973.

"***Factors affecting cricket ball swing***", Mehta, R. D., Bentley, K., Proudlove, M., Varty, P., Nature, 303: pp. 787-788, 1983.

"***Aerodynamics of a cricket ball***", Sherwin, K., Sproston, J. L., Int. J. Mech. Educ. 10: pp. 71-79. 1982.

http://web.archive.org/web/20071018203238/http://www.geocities.com/k_achutarao/MAGNUS/magnus.html#mehta

Olympic – HMS Hawke collision: 20 September 1911, off the Isle of Wight. Large displacement of water by Olympic sucked in the *Hawke* into her side. One crew member of the Olympic, Violet Jessop, survived the collision with the *Hawke*, and also the later sinking of *Titanic* , and the 1916 sinking of *Britannic*, the third ship of the class.



The Hole in the "Olympic," the Damage Below the Waterline being Much Greater Than That Above

The Bow of the "Hawke," the Damage being so Great That the Ram Has Been Mashed Flat

"Popular Mechanics" Magazine December 1911

http://en.wikipedia.org/wiki/File:Hawke_-_Olympic_collision.JPG

http://en.wikipedia.org/wiki/RMS_Olympic

Next: Unit 10

Classical Electrodynamics

Charles
Coulomb
1736-1806

Carl Freidrich
Gauss
1777-1855

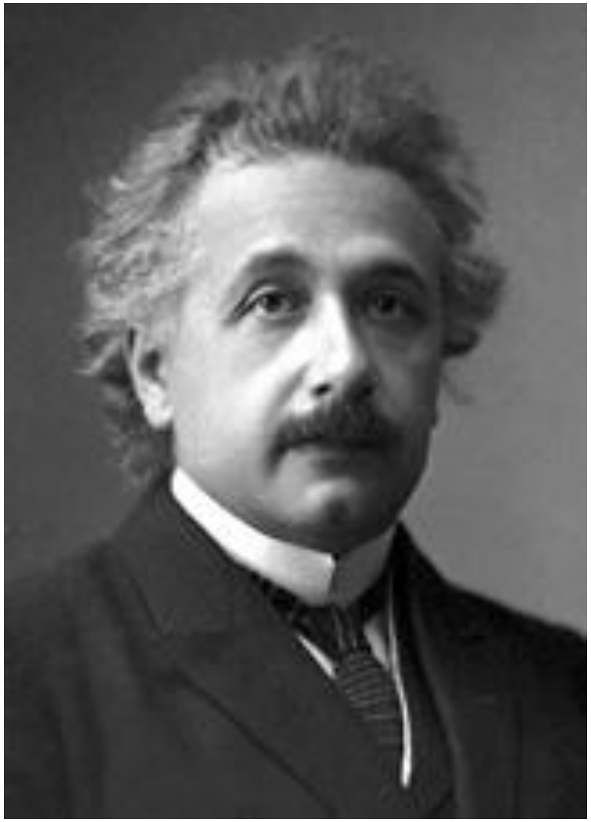
Andre Marie
Ampere
1775-1836

Michael
Faraday
1791-1867

Electrodynamics & STR

The special theory of relativity is intimately linked to the general field of electrodynamics. Both of these topics belong to 'Classical Mechanics'.

James Clerk Maxwell
1831-1879



Albert Einstein
1879 - 1955

$$\vec{\nabla} \bullet \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \bullet \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \frac{\partial \vec{J}}{\partial t} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

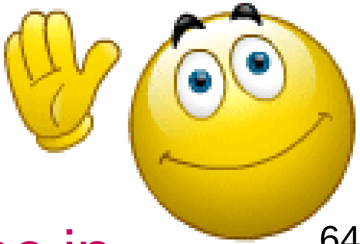
$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

James Clerk Maxwell
1831-1879

Divergence and Curl of $\left(\vec{E}, \vec{B}\right)$

We shall take a break here.

Questions ? Comments ?



Helmholtz Theorem

Curl & Divergence;
+ boundary conditions

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